An Image-Based Approach to Special Relativistic Rendering

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Abstract

This paper describes a novel rendering technique for special relativistic visualization. It is an image-based method which allows to render high speed flights through real-world scenes filmed by a standard camera. The relativistic effects on image generation are determined by the relativistic aberration of light, the Doppler effect, and the searchlight effect. These account for changes of apparent geometry, color, and brightness of the objects. It is shown how the relativistic effects can be taken into account by a modification of the plenoptic function. Therefore, all known image-based non-relativistic rendering methods can easily be extended to incorporate relativistic rendering. Our implementations allow interactive viewing of relativistic panoramas and the production of movies which show super-fast travel. Examples in the form of snapshots and film sequences are presented.

CR Categories: I.3.3 [Computer Graphics]: Picture/Image Generation—Viewing algorithms I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture I.3.8 [Computer Graphics]: Applications—Special relativity J.2 [Physical Sciences and Engineering]: Physics—Theoretical astrophysics

Keywords: image-based rendering, natural phenomena, plenoptic function, scientific visualization

1 Introduction

The visual appearance of fast objects or the world seen from a rapidly moving vehicle is of great interest for the movie and TV industry due to the increasing number of science fiction productions. Einstein's special theory of relativity is capable of describing objects moving with velocities comparable to the speed of light. However, movie production has never been guided by special relativity, but only by artistic issues and technical practicability.

One important reason for this is that the already known special relativistic rendering techniques are not very well suited for film production. They use a standard geometry-based representation of three-dimensional scenes and thus require time-consuming geometrical modeling and costly rendering.

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In this paper, we propose a novel, image-based approach to special relativistic rendering. This approach overcomes problems of geometry-based rendering and has the following important advantages: No three-dimensional geometric modeling is needed, rendering costs are negligible, and photo-realism is easily achieved. Moreover, the examples shown at the end of the paper reveal impressive visual effects due to special relativity. Therefore, the physics-based rendering of spaceships and other super-fast vehicles becomes feasible—both artistically and technically.

A further field of application is the visualization of special relativity for pedagogical reasons. Special relativity is widely regarded as a difficult and hardly comprehensible theory, mainly because the properties of space, time, and light in relativistic physics are totally different from those in classical, Newtonian physics. In many respects, they are contrary to human experience and everyday perception, which is based on low velocities.

In the real world, mankind is limited to very small velocities compared to the speed of light. For example, the speed of light is a million times faster than the speed of an airplane and 40,000 times faster than the speed at which the Space Shuttle orbits the Earth. Even in the long term future, there is no hope of achieving velocities comparable to the speed of light. Therefore, visualization is the only means of directly exploring the realm of special relativity and can thus help the intuition and motivation of people interested in the theory.

The basic idea of the image-based approach to relativistic rendering is presented in Sect. 3. We show how all relativistic effects on image generation can be covered by a modification of the plenoptic function[1]. Therefore, the full three-dimensional information about the scene is not required for relativistic rendering. In this framework, only one additional step is appended to the normal non-relativistic rendering pipeline, which is otherwise left unchanged. Therefore, the relativistic transformation can easily be incorporated in all known image-based rendering methods.

We present two implementations of image-based relativistic rendering. The first implementation is an interactive panorama viewer which creates snapshots of a panorama with the camera moving at arbitrary speed. The second implementation is a batch job-oriented tool for the production of relativistic movies playing in real-world scenes. It stitches and blends series of views taken by different cameras in order to generate a sequence of images for a relativistic flight.

2 Previous and Related Work

Remarkably, the issue of visual perception in special relativity was ignored for a long time, or wrong interpretations were given. The first solutions to this problem were described by Penrose[20] and Terrell[27] in 1959. Later, more detailed descriptions of the geometrical appearance of fast moving objects were given by Weisskopf[30], Boas[2], Scott and Viner[24], and Scott and van Driel[23].

Hsiung and Dunn[12] were the first to use advanced visualization techniques for image shading of fast moving objects. They proposed an extension of normal three-dimensional ray tracing for image shading of fast moving objects. This technique accounts for relativistic effects on the apparent geometry as seen by the observer. Hsiung et al.[13] investigated relativistic ray tracing in more detail and included the visualization of the Doppler effect.

Hsiung et al.[14] introduced the time-buffer for fast visualization of relativistic effects. The time-buffer technique resembles the normal z-buffer and can be mapped onto it. The time-buffer technique allows for relativistic polygon rendering, a scan-line method. It is based on the apparent shapes of objects as seen by a relativistic observer. Gekelman et al.[8] and Chang et al.[3] investigated the polygon rendering approach in detail and gave a comprehensive presentation.

Weiskopf[28] introduced texture-based relativistic rendering for visualizing the apparent geometry of fast moving objects. This approach performs the relativistic transformation on the image plane by texture mapping.

A lot of research has been conducted on the field of non-relativistic image-based rendering. *QuickTime VR*[4] is a well-known method for image-based rendering which uses panorama pictures. More advanced techniques include plenoptic modeling[17], light fields[15], the lumigraph[11], view morphing[25], and hybrid geometry and image-based rendering[5].

3 Basic Idea

One basic feature of special relativity is the absence of a single universal frame of reference and of a universal time. Any inertial frame is equally valid to describe the physical world.

Often an egocentric point of view is adopted to derive the properties of relativistic rendering, i.e. the camera is at rest and the objects are moving. In this paper, we rather take an exocentric point of view. Here, the objects are considered to be at rest and the observer—the camera—is moving at high speed.

The essence of all image-based rendering methods is the evaluation of the plenoptic function[1]. The full plenoptic function $P(x, y, z, t, \theta, \phi, \lambda)$ is the radiance of the light depending on the direction (θ, ϕ) in spherical coordinates, the spatial position (x, y, z), the time t, and the wavelength λ. The definition of wavelength dependent radiance can be found, e.g., in [10, Chapt. 13]. Polarization is usually neglected.

We restrict ourselves to a static world in which all objects and light sources are at rest relative to each other and relative to the objects' frame denoted S_{obj}. In S_{obj}, the plenoptic function can be determined by standard image-based rendering algorithms, since the finite speed of light can be neglected in this static situation.

First, consider the generation of a snaphot taken by a camera at rest in S_{obj}. The spatial position of the camera is (x, y, z) and the time is t. All the information needed for this snapshot is contained in the reduced three-parameter plenoptic function $\tilde{P}(\theta, \phi, \lambda)$ which is evaluated at the respective position and time.

Then, let us bring special relativity back into the game. Consider another observer that is moving relative to the objects. His or her rest frame is denoted S_{observer}. This observer is taking a snapshot at the same position and time as the first observer that is at rest in S_{obj}. What is the plenoptic function for this moving observer and how is it connected to the plenoptic function for the observer at rest?

In general, physical properties can be transformed from one frame of reference to another by the so-called Lorentz transformation. Here, all relevant physical properties are contained in the plenoptic function. Therefore, only the Lorentz transformation of the plenoptic function has to be known. This transformation is discussed in the next section.

Once the plenoptic function $\tilde{P}(\theta, \phi, \lambda)$ with respect to S_{obj} is transformed to $\tilde{P}'(\theta', \phi', \lambda')$ with respect to S_{observer}, the normal rendering process can generate the image seen by the fast moving camera because $\tilde{P}'(\theta', \phi', \lambda')$ is the plenoptic function at rest relative to this camera. (The primed quantities are with respect to S_{observer}.) Therefore, all relativistic effects are isolated in the form of the Lorentz transformation of the plenoptic function. The locality property of this transformation allows us to generate relativistic images without knowledge of the depth, or three-dimensional, information about the surrounding scene. Due to the relativity principle the transformation of the plenoptic function can account for both a fast camera and rapidly moving objects.

4 Lorentz Transformation

In this section, the Lorentz transformation of the plenoptic function is described. Relevant for this transformation are the relativistic aberration of light, the Doppler effect, and the searchlight effect. For a detailed presentation of special relativity we refer to [18, 19, 22].

The relativistic aberration of light causes a rotation of the direction of light when one is changing from one inertial frame of reference to another. The aberration of light is sufficient to completely describe the apparent geometry seen by a fast moving camera. Fig. 1 illustrates the aberration of light.

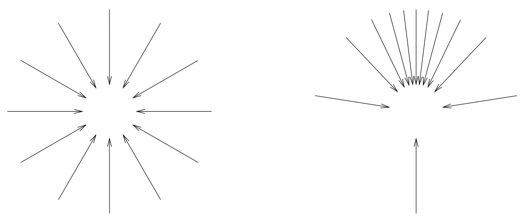

Figure 1: Relativistic aberration of light. The left image shows some of the light rays registered by an observer at rest. The right image shows the same light rays with the observer moving upwards at 90 percent of the speed of light.

The Doppler effect accounts for the transformation of wavelength from one inertial frame of reference to another and causes a change in color.

The searchlight effect is based on the transformation of wavelength dependent radiance from one inertial frame of reference to another. The transformation of radiance increases the brightness of objects ahead when the observer is approaching these objects at high velocity.

Let us consider two inertial frames of reference, S and S', with S' moving with velocity v along the z axis of S. The usual Lorentz transformation along the z axis connects frames S and S'.

In reference frame S, consider a light ray with the direction (θ, ϕ) and the wavelength λ. In frame S', the light ray is described by the direction (θ', ϕ') and the wavelength λ'. These two representations are connected by the expressions for the relativistic aberration of light, cf. [19],

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}, \quad (1)$$

$$\phi' = \phi, \quad (2)$$

and for the Doppler effect,

$$\lambda' = D\,\lambda. \quad (3)$$

The Doppler factor D is defined as

$$D = \frac{1}{\gamma(1 - \beta\cos\theta)} = \gamma(1 + \beta\cos\theta'),$$

where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = v/c$, and c is the speed of light.

Wavelength dependent radiance L_λ is transformed from one frame of reference to another according to

$$L'_\lambda(\lambda', \theta', \phi') = D^{-5} L_\lambda(\lambda, \theta, \phi).$$

A derivation of this relation can be found in [29].

The relativistic aberration of light, the Doppler effect, and the searchlight effect can be combined to form the transformation of the plenoptic function from S to S':

$$\begin{aligned} \tilde{P}'(\theta', \phi', \lambda') &= D^{-5}\tilde{P}(\theta, \phi, \lambda) \\ &= D^{-5}\tilde{P}\left(\arccos\frac{\cos\theta' + \beta}{1 + \beta\cos\theta'}, \phi', \frac{\lambda'}{D}\right) \quad (4) \end{aligned}$$

By inverting Eqs. (1) to (3), the parameters θ, ϕ, and λ have been substituted by terms containing θ', ϕ', and λ'.

Usually, the direction of motion is not identical to the z axis. Therefore, additional rotations of the coordinate system have to be considered before and after the aberration transformation. These rotations are identical to the standard rotations in three-dimensional Euclidean space. By including these rotations, we obtain the complete Lorentz transformation of the plenoptic function.

With the notation from the previous section, the frame S coincides with S_{obj} and the frame S' with S_{observer}. Please note that the transformed plenoptic function depends only on the original plenoptic function and on the observer's velocity and direction of motion.

5 Relativistic Rendering

Image-based relativistic rendering extends the standard non-relativistic techniques by a transformation of the plenoptic function according to the previous section. This extension is located at the end of the rendering pipeline, just before the final image is generated. All other parts of the rendering pipeline are unaffected.

In the following, some variations of relativistic rendering are described. In particular, they address the issue of missing data, since the wavelength dependency of the plenoptic function can usually not be measured. In most cases, data for image-based rendering is acquired by cameras which are sensitive to only three RGB colors and not to the full power spectrum of the incoming light.

5.1 Completely Relativistic Rendering

If the wavelength dependent plenoptic function $\tilde{P}(\theta, \phi, \lambda)$ is provided in the non-relativistic situation, the transformed plenoptic function $\tilde{P}'(\theta', \phi', \lambda')$ can be computed according to Sect. 4. It is important that $\tilde{P}(\theta, \phi, \lambda)$ is known for an extended range of wavelengths, so that $\tilde{P}'(\theta', \phi', \lambda')$ can be evaluated for wavelengths in the visible range after Doppler-shifting.

Each pixel on the image plane has corresponding spherical coordinates (θ', ϕ'), which are transformed to (θ, ϕ) in the objects' frame. Therefore, a wavelength dependent radiance $L'_\lambda(\lambda')$ can be associated with each pixel. For the final display on the screen, three tristimulus values such as RGB have to be calculated from this wavelength dependent radiance. The RGB values (c_R, c_G, c_B) can be obtained by

$$c_i = \int L'_\lambda(\lambda')\bar{f}_i(\lambda')\,d\lambda', \qquad i = R, G, B,$$

where $\bar{f}_i$ are the respective color matching functions for RGB, cf. [32].

5.2 Apparent Geometry

The relativistic effects on the apparent geometry can be visualized by using only a partial transformation of the plenoptic function. Here, merely the effects of the aberration of light are taken into account and the searchlight and Doppler effects are neglected, i.e. only the direction (θ, ϕ) of the incoming light is transformed and all other effects are ignored.

This visualization technique is useful when the full spectral information of the plenoptic function is not available, since this information is not needed for the visualization of apparent geometry. Nevertheless, even this restricted relativistic rendering provides some insight into the special theory of relativity and creates impressive visual effects, as shown in Sect. 7.

5.3 Reconstruction of the Power Spectrum

In most cases, data for image-based rendering does not comprise the full power spectrum, but only three RGB values. The power spectrum has to be reconstructed from RGB values in order to include the relativistic effects on geometry and illumination. Unfortunately, this reconstruction is not unique because infinitely many spectra map to one RGB triplet. This phenomenon is called metamerism, cf. [32].

However, a possible spectrum can always be determined and metamerism gives a lot of freedom of doing so. A straightforward approach models the three RGB values by the line spectrum consisting of the corresponding primaries[9]. Sun et al.[26] propose the representation by Gaussian functions with adapted width. Another approach uses Fourier functions[9].

We find the dominant wavelength model[7] useful because it provides a smooth change of color and brightness for a wide range of Doppler factors. The corresponding spectral power distribution consists of a spike at the dominant wavelength and of a uniform distribution, i.e. white light. The luminance and excitation purity determine the levels of the two parts of the spectrum. The parameters for the dominant wavelength model can be computed from RGB values according to [7]. The relativistic situation requires only one slight extension of the original model. Here, the uniform part of the spectrum is not restricted to the range of visible wavelengths, but comprises a larger interval. In this way, the spectrum is still present after Doppler-shifting.

With the reconstructed wavelength dependent plenoptic function, the algorithm from Sect. 5.1 can generate the fully relativistic image.

5.4 Rendering of a Film Sequence

So far, the generation of just a single snapshot has been described. But how can a film sequence with a fast camera be produced?

In principle, it works the same way as in the non-relativistic situation. The path of the camera is discretized into a finite set of positions. For every element of this set the plenoptic function is evaluated. Therefore, the plenoptic function has to be known at these positions. Then, the relativistic transformation is computed and the corresponding image is generated. Finally, a list of snapshots which represent the film sequence is obtained.

For the film to be physically sound, not just the generation of each single snapshot has to be correct, but also the path of the camera itself. As long as the camera is moving uniformly—at constant speed and with a constant direction of motion—the camera is trivially placed at equidistant positions. However, even an accelerated camera can be described by special relativity. In [21] it is shown how the trajectory of an accelerating observer can be computed. Therefore, the positions and velocities of the camera for each snapshot can be calculated, and image-based relativistic rendering can be performed. This method is valid because the generation of a single image is only determined by the position and velocity of the viewer and by the standard camera parameters, but not by the "history" of the trajectory or the acceleration of the observer.

6 Implementation

We have implemented the relativistic panorama viewer *Imagine* (IMAge-based special relativistic render enGINE), which can read panoramas in the *LivePicture* format[16]. This format is similar to *QuickTime VR*, but uses a spherical projection instead of a cylindrical projection. Therefore, a complete 4π sterad view is supported.

The interactive viewer is written in C++ and is based on standard OpenGL 1.1[31]. The virtual camera is surrounded by a sphere onto which the panorama texture is mapped. Texture mapping hardware is used to achieve high rendering performance. The relativistic effects on the apparent geometry are implemented by transforming the texture coordinates according to the relativistic aberration of light. The aberration of light changes the solid angle—objects in the direction of motion are scaled down and those in the back become magnified. Therefore, the resolution of the original panorama has to be high enough to guarantee a relativistic image of good quality. The non-interactive part of the viewer uses software rendering to implement completely relativistic visualization by reconstructing the spectrum according to Sect. 5.3.

Another implementation is *Off-Terdingen*, which is an off-screen, batch job-oriented relativistic movie renderer. It is able to produce movies of relativistic flights through real-world scenes. It is a C++-based software renderer which stitches and blends series of views taken by different cameras in order to generate a sequence of images for a relativistic flight. The parameters and orientations of the cameras are supplied manually.

7 Results

Figs. 2 through 4 provide examples of image-based relativistic rendering. These images have been produced by *Imagine*. The orginal data for the non-relativistic panorama was taken from [6].

Fig. 2 shows the non-relativistic view of the scene. Fig. 3 illustrates the effects on apparent geometry when the viewer is rushing into the scene with $\beta = 0.9$. A dominant effect is the increased apparent field of view—the objects seem to move away. Furthermore, straight lines which are perpendicular to the direction of motion become distorted to hyperbolae, which is prominent for the texture on the floor.

Fig. 4 shows completely relativistic rendering with $\beta = 0.3$. Here, the power spectrum is reconstructed by using the dominant wavelength model. Changes in color and brightness due to the Doppler and searchlight effects are noticeable. The searchlight effect heavily brightens the image. Therefore, the overall intensity has to be reduced to one half of that in Figs. 2 and 3 in order to avoid extreme clamping of the final RGB values.

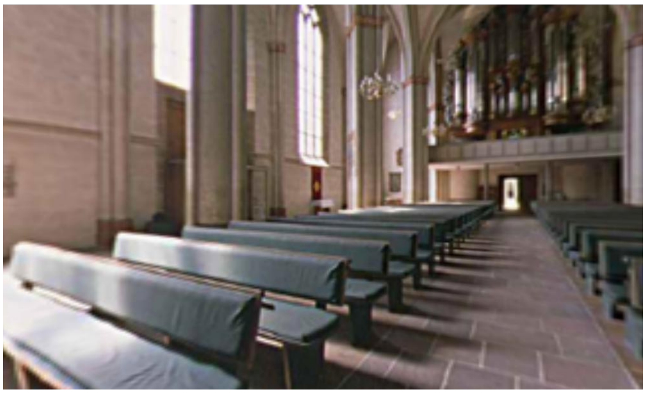

Figure 2: Non-relativistic view.

Figure 3: Relativistic visualization of apparent geometry with $\beta = 0.9$.

The figure on the first page shows the apparent geometry for a snapshot of the Yosemite Valley at $\beta = 0.95$.

The accompanying video presents a relativistic trip across a bridge. This movie has been produced with the use of *Off-Terdingen*. Here, the recordings of three cameras have been stitched together to form the final movie. The movie was broadcast on TV as part of an edutainment show on relativity.

8 Conclusion and Future Work

In this paper an image-based approach to special relativistic rendering has been introduced. This approach closes the gap between the well-known non-relativistic image-based techniques and relativistic visualization. We have shown how all relativistic effects on image generation can be covered by a transformation of the plenoptic function. Therefore, only slight modifications of existing rendering methods are required to incorporate the physically correct rendering of super-fast objects.

Photo-realistic images of rapidly moving real-world objects can be generated with great ease. Therefore, image-based special relativistic rendering is a powerful tool to generate movies and snapshots for both entertainment and educational purposes.

In future work, we will improve the techniques for data acquisition. In particular, we will build a robot-based camera system which can automatically film a 4π sterad field of view. The resolution will be adapted to the different scalings due to aberration in

Figure 4: Completely relativistic rendering with $\beta = 0.3$. The overall intensity is reduced to one half of that in Figs. 2 and 3 in order to avoid extreme clamping of the final RGB values.

order to achieve high-quality final images.

Acknowledgements

This work was supported by the Deutsche Forschungsgemeinschaft (DFG) and is part of the project D4 within the Sonderforschungsbereich 382.

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